Effects of Doppler Rate on Subcarrier Demodulator Assembly Performance

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A time dependent, steady-state static phase error equation that includes doppler rate effects has been incorporated into the Subcarrier Demodulator Assembly efficiency program. With the program in its present form, the optimum possible bandwidths, bit rate, amount of nulling of static phase error, and degradation expected can be obtained for the tracking of a given two-way doppler shift profile by various manipulations of the program.

I. Introduction

The presence of significant doppler rate in future DSN missions raises the following questions with respect to Subcarrier Demodulator Assembly (SDA) performance:

(1) What loop bandwidths should be used for the various data rates?

(2) How often should the SDA static phase error (SPE) be nulled?

(3) What degradation should be expected for periods of significant doppler rate?

This article presents analysis to help answer the above questions. What follows is the derivation of a steady-state, time dependent, static phase error equation that includes error terms due to doppler rate and higher order effects. A computer program is then presented that incorporates the new static phase error equation and finally an example of the use of the computer program is given using Pioneer 10 data.

II. Derivation of Static Phase Error Equation

Using the linear SDA model as presented in Brockman (Ref. 1) (Fig. 1) the error ratio is given by:

$$\frac{E(s)}{\Theta(s)} = \frac{1}{1 + \dfrac{G}{s}\left(\dfrac{1+\tau_2 s}{1+\tau_1 s}\right)} = \frac{s + \tau_1 s^2}{G + (\tau_2 G + 1)s + \tau_1 s^2} \quad (1)$$

from which, if we perform a series expansion by dividing the numerator polynomial by the demoninator polynomial, we obtain:

$$\frac{E(s)}{\Theta(s)} = C_0 + C_1 s + C_2 s^2 + C_3 s^3 + \cdots \quad (2)$$

Therefore:

$$E(s) = C_0\Theta(s) + C_1 s\Theta(s) + C_2 s^2\Theta(s) + C_3 s^3\Theta(s) + \cdots \quad (3)$$

Neglecting initial conditions and impulses at $t = 0$, we obtain the steady state error equation:

$$E_{ss}(t) = C_0\Theta(t) + C_1\Theta'(t) + C_2\Theta''(t) + C_3\Theta'''(t) + \cdots \tag{4}$$

Assuming contributions of higher order terms are negligible, the steady-state static phase error equation is:

$$E_{ss}(t) \cong C_0\Theta(t) + C_1\Theta'(t) + C_2\Theta''(t) + C_3\Theta'''(t) + C_4\Theta^{iv}(t) \tag{5}$$

Evaluation of Eq. 1 gives the following values for C_i's:

$$C_0 = 0$$

$$C_1 = \frac{1}{G}$$

$$C_2 = \frac{G(\tau_1 - \tau_2) - 1}{G^2}$$

$$C_3 = \frac{2G(\tau_2 - \tau_1) + G^2(\tau_2^2 - \tau_1\tau_2) + 1}{G^3}$$

$$C_4 = \frac{-(\tau_2 G + 1)^3 + (2\tau_1(\tau_2 G + 1) + \tau_1(\tau_2 G + 1)^2)G}{G^4} - \frac{\tau_1^2 G}{G^4}$$

since $C_0 = 0$,

$$E_{ss}(t) \cong C_1\Theta'(t) + C_2\Theta''(t) + C_3\Theta'''(t) + C_4\Theta^{iv}(t) \tag{6}$$

where

$E_{ss}(t)$ = steady-state static phase error

$\Theta'(t)$ = input doppler shift function

$\Theta''(t)$ = input doppler rate function

$\Theta'''(t), \Theta^{iv}(t)$ = input higher order effects

Equation 6 gives the steady-state static phase error as a function of time for a given doppler shift curve and its respective derivatives, $\Theta'(t)$, $\Theta''(t)$, etc.

Close agreement is found between Eq. 6 and published results by Tausworthe (Ref. 2) for second order loops with doppler shift and doppler rate inputs. For example, if the input phase function with phase offset Θ_0 and frequency offset Ω_0 is:

$$\Theta = \Theta_0 + \Omega_0 t$$

then Eq. 6 predicts, for a passive integrator, second order loop, the steady-state static phase error to be:

$$E_{ss}(t) = \frac{\Omega_0}{G}$$

which is the same value or that predicted by Tausworthe. For an input phase function with doppler rate Λ_0 given by:

$$\Theta = \Theta_0 + \Omega_0 t + \frac{\Lambda_0 t^2}{2}$$

Eq. 6 predicts, for a passive integrator, second order loop, the steady-state static phase error to be ($\tau_1 >> \tau_2$):

$$E_{ss}(t) \cong \frac{(\Omega_0 + \Lambda_0 t) + \tau_1\Lambda_0}{G}$$

This compares favorably with Tausworthe's result:

$$E_{ss}(t) \cong \frac{(\Omega_0 + \Lambda_0 t)}{G}$$

Two assumptions in the above analysis were made. First, it is assumed that the static phase error can be described using a steady-state equation. This assumes the SDA loop dynamics are such that any transients introduced into the loop die out quickly. Second, the above analysis assumes that the steady-state error equation can be approximated by four terms, which implies that the error series converges rapidly. For the doppler data used for Pioneer this is a good assumption. However, for another doppler profile and different subcarrier/carrier ratios, this assumption would have to be further tested.

III. Computer Program

To use the static phase error equation as given by Eq. 6 in the current SDA efficiency program, a curve fit of doppler shift predict data must first be obtained. The coefficients, $a_0, a_1, a_2, \cdots$ of a curve fitting polynomial of the form

$$y = a_0 + a_1 x + a_2 x^2 + \cdots a_n x^n$$

are then inputted into the program in ascending order. At least 5 coefficients, starting with a_0 must be specified.

If fewer than five coefficients are necessary to describe the fitted polynomial, then the remaining coefficients must be inputted as

$$a_i = 0_{i=n,4}$$

The current program is set up to accept only two-way doppler shift profiles given in MHz with time scales given in minutes.

The following information is supplied by the user to the SDA efficiency program:

(1) Data bit rate (bps), BRATE.

(2) Number of bits/symbol for data, RN.

(3) The time interval of interest for doppler study (minutes), LTMIN and LTMAX.

(4) Sampling time interval (minutes), INCRE.

(5) The maximum allowable degradation in the SDA (dBs), ADMAX.

(6) The modulation index (radians), TMOD1.

(7) The system temperature (Kelvin), TEMP.

(8) The appropriate carrier and subcarrier frequencies (Hz), FCAR and FSUBC.

(9) The carrier power (dBM), PCDBM.

(10) SDA loop design bandwidth (Hz), I = narrow, medium, or wide.

(11) The doppler shift curve fitted polynomial coefficients, $a_0, a_1, \cdots, a_n$.

The program computes, first nulling out the static phase error at time LTMIN, all static phase error for times LTMIN to LTMAX at increments of time INCRE. The overall static phase error (doppler plus phase noise) is then calculated along with SDA degradation,

$$\left(\frac{\text{demod } \dfrac{ST_{sy}}{N_0}}{\text{input } \dfrac{ST_{sy}}{N_0}} \right)$$

in dBs.

The following information is outputted by the program for time increments of INCRE.

(1) Values of doppler shift, DELF1, (Hz) for LTMIN to LTMAX.

(2) Doppler rate, DELF2 (Hz/s), for LTMIN to LTMAX.

(3) Second and third order derivatives values, DELF3 (Hz/s^2), and DELF4 (Hz/s^3).

(4) Time T (min) at which static phase error is computed.

(5) SDA Efficiency

$$\left(\frac{\text{demod } \dfrac{ST_{sy}}{N_0}}{\text{input } \dfrac{ST_{sy}}{N_0}} \right)$$

(6) Degradation as defined in Eq. (5) (phase noise + doppler) in dBs, DEGDB.

(7) Added degradation as defined in Eq. (5) due to doppler alone (dB), DEGDB.

(8) Static phase error due to doppler (radians), SPESDA.

(9) Nulling factor (radians), SPET.

The SPET is the amount of static phase error that has accumulated when the SPE is nulled on the SDA.

The computer deck setup follows:

```
@RUN

@FOR

User supplied information (BRATE,RN,LTMIN,
LTMAX,INCRE,ADMAX,TMOD1,TEMP,FCAR,
FSUBC,PCDBM,I=NARROW,MEDIUM, or WIDE)

PROGRAM

END

@MAP

LIB LIB*JPL$

@XQT

±X.XXXXXX±XX ← DATA FORMAT  )
   .                        ) Doppler
   .  (at least 5 coefficients) ) profile
   .                        ) coefficients
   .                        )
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From the computer program the number of times the SDA SPE must be nulled for some ADMAX is easily found by counting the number of times during tracking the SPE is nulled so SDA degradation is never greater than ADMAX.

The user can obtain the maximum amount of degradation expected for a tracking period without constant nulling by setting ADMAX to some arbitrarily large number that would never occur during a tracking period. For example set ADMAX = 100.

The SDA efficiency program can be used to select optimum design bandwidths for a given doppler profile as a function of time by iterating over the set of available bandwidths for a given bit rate and carrier power, then storing the degradation as a function of time, and finally searching the stored values for optimum bandwidths.

IV. Example of the Computer Program—Pioneer 10

Using the SDA efficiency program with the new static phase error equation, a doppler shift curve was obtained for Pioneer 10, DSS 12 predicts for a Jupiter encounter. Using a least squares polynomial fit routine, PFIT in the JPL Subroutine Library, an eighth order doppler shift equation was produced for time -420 to $+74$ min before and after encounter. A sampling interval of 5 min was used. A bit rate of 128 bps with narrow bandwidth and a carrier power of -152 dBm was used. ADMAX, the maximum allowable degradation before nulling, was set to 0.1 dB. The above inputs necessitate the nulling of the static phase error three times, and the maximum predicted overall degradation (doppler + phase noise) was 0.16 dB.

References

1. Brockman, M. H., "MMTS: Performance of Subcarrier Demodulator," in *Supporting Research and Advanced Development*, Space Programs Summary 37-52, Vol. II, p. 131, Jet Propulsion Laboratory, Pasadena, Calif., July 31, 1968.
2. Tausworthe, R. C., *Theory and Practical Design of Phase-Locked Receivers*, Technical Report 32-819, Vol. I, p. 36. Jet Propulsion Laboratory, Pasadena, Calif., Feb. 15, 1966.

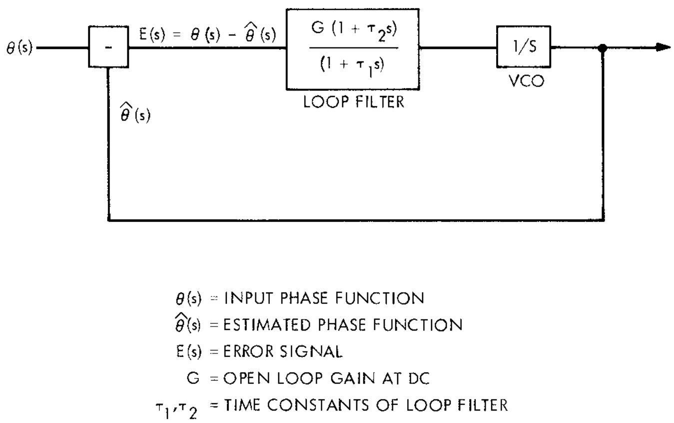

Fig. 1. Linear model of subcarrier phase–tracking loop (Laplace representation)